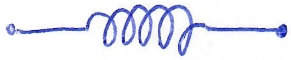
# ME 4555 - Lecture 5 - RLC circuits



We will cover $\underbrace{\text{RLC circuits}}_{\text{this lesson}}$, op-amps, motors.

## Common circuit elements

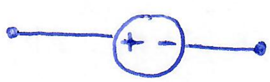
Resistor:  (R) in Ohms ($\Omega$)

Inductor:  (L) in Henrys (H)

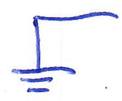
Capacitor:  (C) in Farads (F)

Voltage source: 

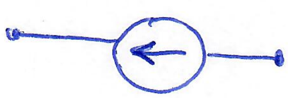
(battery)



(general)

 (ground)

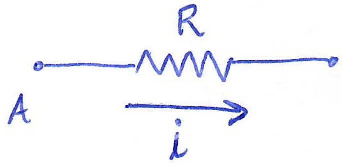
[indicates voltage reference of 0 volts]

Current source: 

Voltage is like "potential energy" in the sense that it is always measured with respect to a reference. Voltage is either a difference between two points or the value of a single point if measured relative to ground.

Current flows from higher voltage to lower voltage.

**Resistors:**




$$i = \frac{1}{R}\underbrace{(V_A - V_B)}$$

current flowing through the resistor

voltage difference, a.k.a. "potential difference" between both terminals. If $V_A > V_B$, current flows right.

Resistors are dissipative elements (energy is lost as heat).

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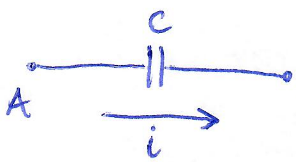
**Capacitors:**




$$i(t) = C\frac{d}{dt}\left(V_A(t) - V_B(t)\right)$$

current stops flowing once the voltage difference between the terminals becomes constant.

An ideal capacitor stores energy, like a battery.

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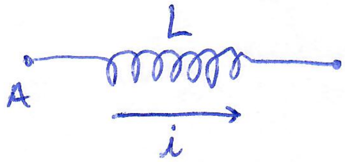
**Inductors:**




$$V_A(t) - V_B(t) = L\frac{d}{dt}i(t)$$

if current flowing is constant, there is no change in voltage (behaves like a short circuit)

An ideal inductor stores energy in a magnetic field.

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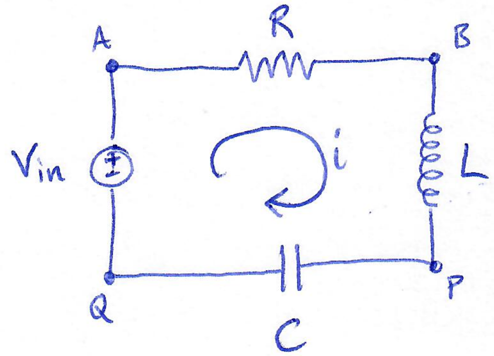
☆ the capacitor formula is often written in "integral form" as:

$$V_A(t) - V_B(t) = \frac{1}{C}\int i(t)\,dt.$$

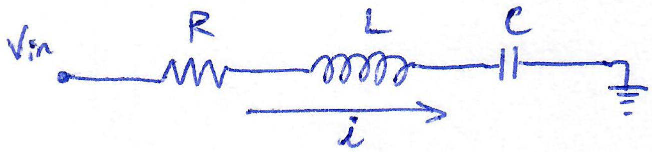
# RLC circuit




Current flows clockwise because it flows from "+" terminal of voltage source to "−" terminal.

$V_{in}$ refers to the difference in voltage between A and Q, i.e. $V_A - V_Q = V_{in}$.

We can set, for example, $V_Q = 0$ and then measure all other voltages with respect to this reference, i.e. $V_A = V_{in}$, etc. In this case, an equivalent diagram would be:




Elements are in series, so $i(t)$ is the same for all three elements.

We have, from the definitions on the prev. page:

$$
\left.
\begin{aligned}
&V_A - V_B = iR. \\
&V_B - V_P = L\frac{di}{dt} \\
+\quad &V_P - V_Q = \frac{1}{C}\int i\,dt \\
&V_{in} = V_A - V_Q
\end{aligned}
\right\} \text{ sum them together}
$$

$$
\boxed{V_{in} = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt}
$$

this is like an EOM. It shows how changing $V_{in}(t)$ is related to the current $i(t)$ through the circuit.

# RLC, cont'd

$$L \frac{di}{dt} + Ri + \frac{1}{C}\int i\, dt = V_{in}. \qquad (1)$$

Current is the rate of change of charge: $i(t) = \frac{d}{dt} q(t)$.

Substituting this into (1), we obtain:

$$\boxed{L\ddot{q} + R\dot{q} + \frac{1}{C} q = V_{in}}$$

Compare this to a spring-mass-damper EOM:

$$\boxed{m\ddot{x} + b\dot{x} + kx = f}$$

Note that $R \leftrightarrow b$ is the damping (energy dissipation) term whereas $L$ and $C$ are analogous to $m$ and $\frac{1}{k}$.

Another interpretation: if $V_{in}$ is constant, i.e. $\frac{d}{dt} V_{in} = 0$, then we can differentiate (1) w.r.t. $t$ and obtain:

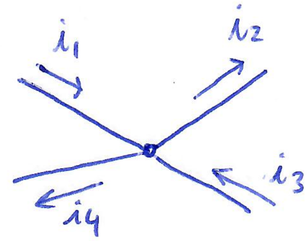
$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = 0$$

Again, same as spring-mass-damper system

Two important rules governing circuits:

## Kirchoff's current law (KCL)

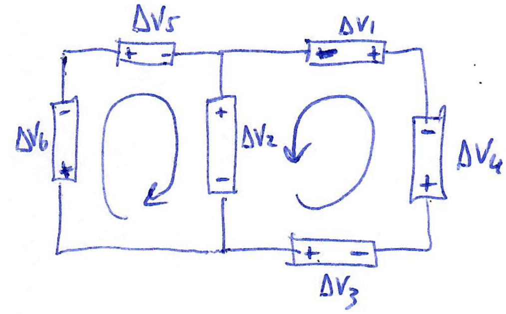
The sum of currents at every junction is equal to zero.




$$i_1 + i_3 = i_2 + i_4$$

## Kirchoff's voltage law (KVL)

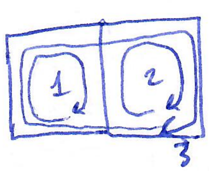
The sum of voltage differences $\Delta V$ across any loop is zero.




$$\begin{cases} \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0 \\ \Delta V_6 + \Delta V_5 + \Delta V_2 = 0 \\ \Delta V_6 + \Delta V_5 - \Delta V_1 - \Delta V_4 - \Delta V_3 = 0 \end{cases}$$

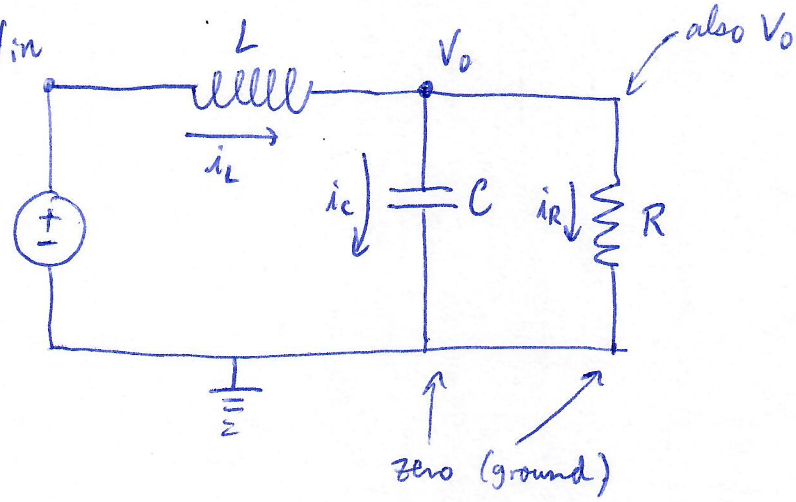
Writing out KCL for each junction and KVL for each loop provides all information available in the circuit.

<u>Note</u>: for KVL, only need to count loops that involve each path at most twice. e.g.




there are 3 loops here, but only 2 of them are needed (see in example above; third KVL eqn is redundant and follows from the first two).

Ex:




**Question**: how is the output voltage $V_o$ related to the input voltage $V_{in}$?

inductor: $$V_{in} - V_o = L\frac{di_L}{dt} \qquad (1)$$

capacitor: $$V_o = \frac{1}{C}\int i_c\,dt. \qquad (2)$$

resistor: $$V_o = R\, i_R \qquad (3)$$

KCL @ $V_o$: $$i_L = i_c + i_R \qquad (4)$$

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Substitute (1), (2), (3) into (4): (to eliminate $i_L, i_c, i_R$)

$$\underbrace{\frac{1}{L}\int (V_{in} - V_o)\,dt}_{i_L} = \underbrace{C\dot{V}_o}_{i_c} + \underbrace{\frac{1}{R}V_o}_{i_R}.$$

Differentiate both sides:

$$\frac{1}{L}(V_{in} - V_o) = C\ddot{V}_o + \frac{1}{R}\dot{V}_o$$

Re-arrange:

$$\boxed{LC\ddot{V}_o + \frac{L}{R}\dot{V}_o + V_o = V_{in}}$$

Again, this looks like a spring-mass-damper system, but slightly different than the series RLC circuit from p. 3-5.